Proving Statements about Segments

Goals
- Justify statements about congruent segments.
- Write reasons for steps in a proof.

VOCABULARY

Theorem a statement that can be proven

Two-column proof has numbered statements and corresponding reasons that show an argument in logical order

THEOREM 2.1 PROPERTIES OF SEGMENT CONGRUENCE

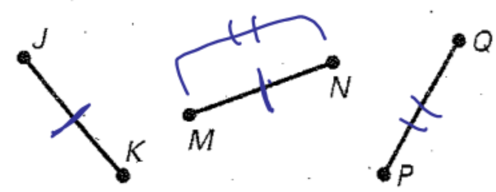
Reflexive For any segment AB, $AB = AB$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 1 ***Transitive Property of Segment Congruence***

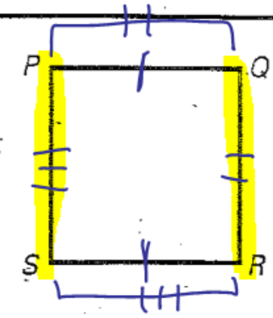
You can prove the Transitive Property of Segment Congruence as follows.



Given: $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$
Prove: $\overline{JK} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$	1. Given
2. $JK = MN$, $MN = PQ$	2. Def. of congruent segments
3. $JK = PQ$	3. Transitive property of equality
4. $\overline{JK} \cong \overline{PQ}$	4. Definition of congruent segments

Example 2 Using Congruence



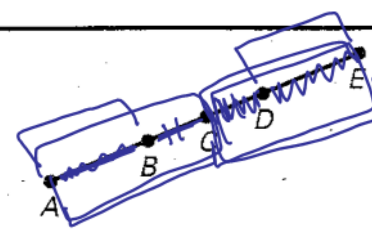
Use the diagram and the given information to complete the proof.

Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PQ} \cong \overline{QR}$, $\overline{PS} \cong \overline{RS}$

Prove: $\overline{PS} \cong \overline{QR}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. Given
3. $\overline{RS} \cong \overline{QR}$	3. Transitive Property of Congruence
4. $\overline{PS} \cong \overline{RS}$ $\overline{RS} \cong \overline{PS}$	4. Given
5. $\overline{PS} \cong \overline{QR}$	5. Transitive Property of Congruence

Example 3 Using Segment Relationships



In the diagram, $AC = CE$ and $AB = DE$. Show that C is the midpoint of $\overline{BD}$.

Solution

Given: $AC = CE$, $AB = DE$

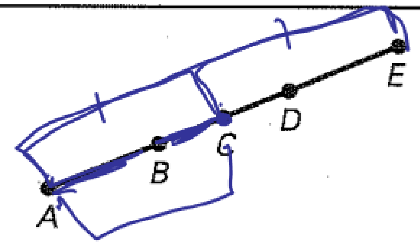
Prove: C is midpoint of $\overline{BD}$

$AC = CE$

$AC = AB + BC$

Statements	Reasons
1. $AC = CE$	1. Given
2. $AB + BC = AC$	2. Segment Add Post
3. $AB + BC = CE$	3. Transitive Property of Equality
4. $CD + DE = CE$	4. Segment Add Post
5. $AB + BC = CD + DE$	5. Transitive Property of Equality
6. $AB = DE$	6. Given
7. $AB + BC = CD + AB$	7. Substitution prop of equality
8. $BC = CD$	8. Subtraction Property of Equality
9. $\overline{BC} \cong \overline{CD}$	9. Definition of congruent segments
10. C is the midpoint of $\overline{BD}$.	10. Def. of midpoint

1. In the diagram, $AB = DE$ and $BC = CD$. Complete the proof to show that C is the midpoint of $\overline{AE}$.



Given: AB=DE, BC=CD

Prove: C is midpoint of AE

Statements	Reasons
1. $AB = DE$	1. Given
2. $AB + BC = DE + BC$	2. Add POE
3. BC = CD	3. Given
4. $AB + BC = DE + CD$	4. Subs
5. $AB + BC = AC$	5. Seg Add Post
6. DE + CD = AC	6. Transitive Property of Equality
7. DE + CD = CE	7. Segment Addition Postulate
8. CE = AC	8. Transitive Property of Equality
9. AC ≅ CE	9. Definition of congruent segments
10. C is the midpoint of $\overline{AE}$.	10. Def of midpoint

2.6 Proving Statements about Angles

Goals
- Use angle congruence properties.
- Prove properties about special pairs of angles.

THEOREM 2.2 PROPERTIES OF ANGLE CONGRUENCE

Angle congruence is reflexive, symmetric, and transitive.

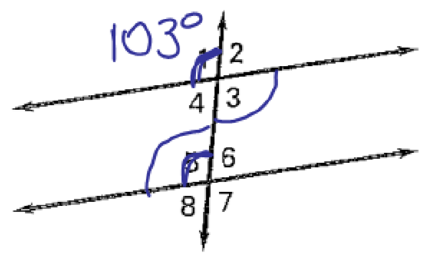
Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Example 1 *Using the Transitive Property*

In the diagram at the right, $\angle 1 \cong \angle 5$, $\angle 5 \cong \angle 3$, and $m\angle 1 = 103°$. What is the measure of $\angle 3$? Explain your reasoning.



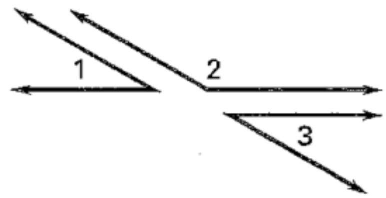
Solution

Statement	reason
1. <1≅<5	1. given
2. <5≅<3	2. given
3. $\angle 1 \cong \angle 3$	3. Transitive property
4. m<1=m<3	4. Def of congruent ∠'s
5. $m\angle 1 = 103°$	5. given
6. m<3=103°	6. transitive

THEOREM 2.3 RIGHT ANGLE CONGRUENCE THEOREM

All right angles are congruent

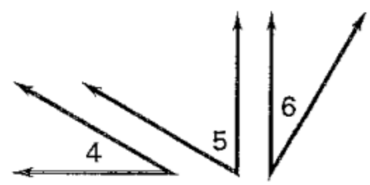
THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM



If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$

THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM



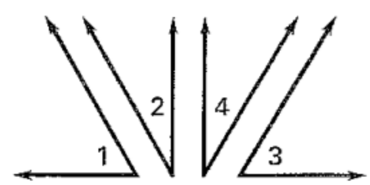
If two angles are complementary to the same angle (or to congruent angles), then the two angles are congruent

If $m\angle 4 + m\angle 5 = 90°$ and $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.

Example 2 ***Proving Theorem 2.5***

Given: $\angle 1$ and $\angle 2$ are complements,
$\angle 3$ and $\angle 4$ are complements,
$\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$

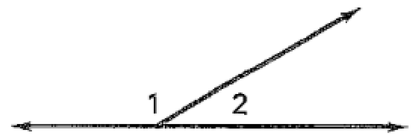


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complements, $\angle 3$ and $\angle 4$ are complements, $\angle 2 \cong \angle 4$	1. given
2. $m\angle 1 + m\angle 2 = 90°$, $m\angle 3 + m\angle 4 = 90°$	2. Def of comp ∡'s
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive property of equality
4. $m\angle 2 = m\angle 4$	4. Def of congruent ∡'s
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	5. Substitution property of equality
6. $m\angle 1 = m\angle 3$	6. Subtraction property of equality
7. $\angle 1 \cong \angle 3$	7. Def of congruent ∡'s

POSTULATE 12 LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are supplementary

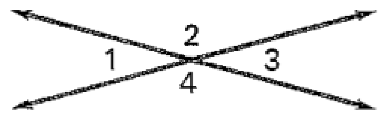
$m\angle 1 + m\angle 2 =$ 180°



THEOREM 2.6 VERTICAL ANGLES THEOREM

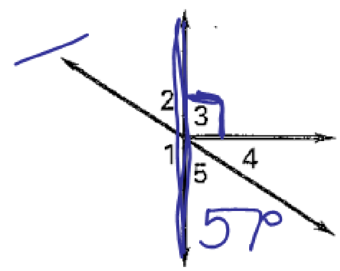
Vertical angles are congruent.

$\angle 1 \cong$ ∠3 and ∠2 $\cong \angle 4$



Example 3 *Using Linear Pairs and Vertical Angles*

In the diagram, $\angle 3$ is a right angle and $m\angle 5 = 57°$. Find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.



Solution

By the definition of a right angle, $m\angle 3 =$ 90°.

$\angle 2$ and $\angle 5$ are vertical angles and $m\angle 5 = 57°$, so $m\angle 2 =$ 57°.

$\angle 1$ and $\angle 5$ form a linear pair, so $m\angle 1 + m\angle 5 =$ 180°. When you substitute 57° for $m\angle 5$ and solve for $m\angle 1$, the result is $m\angle 1 =$ 123°.

$\angle 4$ and $\angle 5$ are complimentary, so $m\angle 4 + m\angle 5 =$ 90°. When you substitute 57 for $m\angle 5$ and solve for $m\angle 4$, the result is $m\angle 4 =$ 33°.